Research on the application of a dynamic mathematical model to the monitoring of an aquatic ecosystem

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Abstract. The monitoring of an aquatic ecological system can be performed using computer engineering. This paper presents an ecological system consisting of a natural lake, a river that feeds the lake and a river which leads the water from the lake to another lake. Using computing and making certain approximations it can be continuously evaluated the depth of the water in the lake depending on the inbound and outgoing flow, an extremely important parameter for the aquatic ecosystem.
Key Words: monitoring, aquatic ecosystem, matematical model.

General aspects. When we need to monitor a complex system, even if it is an aquatic one, we start to establish the basic parameters that must be followed. In this regard, for a complex aquatic eco-system, the parameters that must be followed are partly established based on actual site conditions and partly on the basis of legislation. Starting from this reality, an important parameter, in my opinion, is the flow of water from entering the system and the water flow out of the system. By pursuing these two parameters can be inferred the variation of the system's water volume, respectively the ecosystem water depth, respectively that of the lake.

Water depth is one of the most important parameters upon which the very existence of the ecosystem depends. These parameters are important from the point of view of the possibility of monitoring and modifying them, respectively the flows in and out of the lake, in order to maintain the constant water level in the ecosystem (Rusu 2010).

The following is an example of inferring a dynamic mathematical model for the accumulation of water in an enclosed space, which can be a natural or artificial lake or a vessel with a certain volume, the process being shown in Figures 1 and 2 (Rusu 2010).

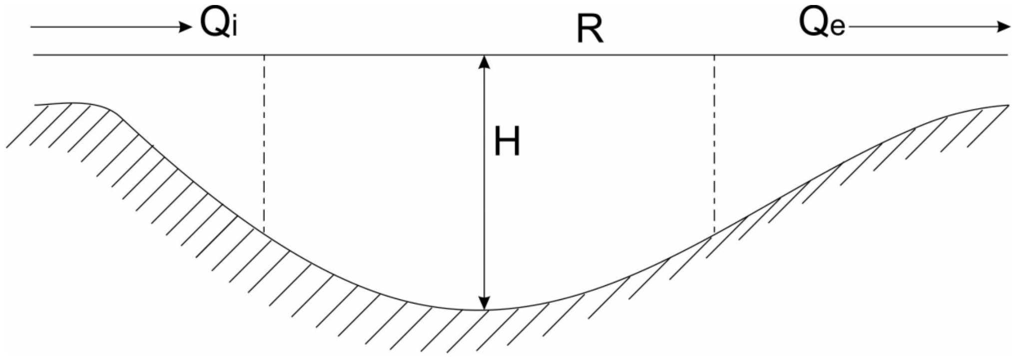

Figure 1. The process of accumulation of a liquid in a space (Qi, Qe – flow of input, output; R – hydraulic resistance; H – current level).

As a simplifying assumption it is assumed that the liquid flow leaving the lake or premises is proportional to the liquid level in the lake or premises, namely:

$$Q_e = k \cdot H \ [1]$$

where *k i*s a coefficient of proportionality.

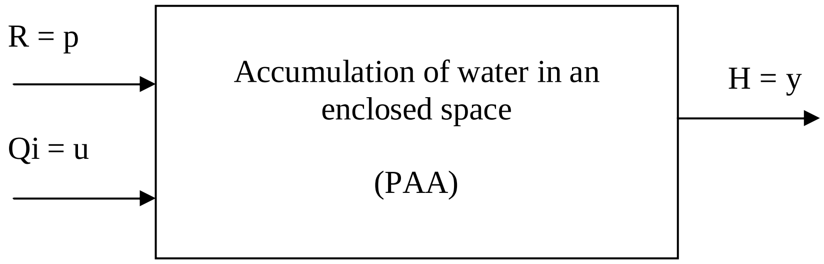

Figure 2. Scheme of the structure of dynamic mathematical model.

As long as the Qi and Qe flow coincide, the liquid level H from the lake remains constant. If equality is no longer observed, the volume of liquid in the lake or site will change with ΔV respectively (Rusu 2011):

$$\Delta V = A.\Delta H \qquad [2]$$

where ΔH is the change in the level of the lake and A is the cross-sectional area of the enclosure.

On the other hand, the volume variation ΔV can be expressed according to the flows:

$$\Delta V = Qi\ \Delta t - Qe\ \Delta t \qquad [3]$$

where Δt is the time interval during which the volume of liquid in the lake or site changes by ΔV .

From the above relations results:

$$A\Delta H = Qi\ \Delta t - Q\ e\ \Delta t\ [4]$$

Or, by dividing through Δt

$$A\frac{\Delta H}{\Delta t} = Q_i - Q_e \qquad [5]$$

Considering the Δt as very small results :

$$\lim_{\Delta t \to 0} \frac{\Delta H}{\Delta t} = \frac{dH}{dt} \qquad [6]$$

By replacing in the initial relation we obtain:

$$A\frac{dH}{dt} = Q_i - kH \qquad [7]$$

or

$$\frac{A}{k}.\frac{dH}{dt} + H = \frac{1}{k}Q_i \qquad [8]$$

Noting $\frac{A}{k} = a$ and $\frac{1}{k} = b$ we obtain

$$a.\frac{dH}{dt} + H = b.Q_i \qquad [9]$$

The dynamic mathematical model thus obtained is represented by a differential equation, linear, inhomogeneous with constant coefficients. The coefficient a of the derivative (expressed in units of time) is called *the time constant* and the coefficient b of the free term is called transfer coefficient (Rusu 2011).

Further on we shall will determine the dynamic characteristic (CD) of the process illustrated in Figure 1, for which the dynamic mathematical model was determined (MMD) (Figure 2).

In specific terms of an aquatic ecosystem, of a lake fed by a river, taking into consideration the following practical data, we obtained:

Q_i = 10 m^3 / h ;
H (0) = 1.5 m (mean baseline)
a = 2 minutes
b = 0.1 m / (m^3/h)
e arrive at the following expression for the dynamic mathematical model (DMM):

$$2\frac{dH}{dt} + H = 0{,}1.10 \qquad [10]$$

respectively :

$$2.dh = (1 - H).dt \qquad [11]$$

or

$$\frac{dH}{1-H} = \frac{1}{2}.dt \qquad [12]$$

by integrating it we obtain :

$$\int_0^H \frac{dH}{1-H} = \frac{1}{2}\int_0^t dt + C \qquad [13]$$

Where C is a constant about to be determined. From the previous relation results that:

$$\ln(1-H) = \frac{1}{2}t + C \qquad [14]$$

or

$$1 - H = e^{-\frac{1}{2}t - C} = e^{-C}.e^{-\frac{1}{2}t} \qquad [15]$$

from where

$$H = 1 - e^{-C}.e^{-\frac{1}{2}t} \qquad [16]$$

But H(0) = 1,5 m , which we replace in the relation above and obtain the following result:

$$1 - e^{-C} = 1{,}5 \qquad [17]$$

respectively :

$$e^{-C} = -0{,}5 \qquad [18]$$

Substituting this result into the equation that defines the dynamic characteristic of the lake level is reached this expression:

$$H(t) = 1 + 0{,}5.e^{-\frac{1}{2}t} \qquad [19]$$

Considering the size H as time constant from the dynamic mathematical model we obtained the "stationary mathematical model", namely:

$$H = b \,.\, Qi \qquad [20]$$

The relation allows us to determine the influence of the inlet flow *Qi* of the lake water level, as follows: Qi (0) = H (0) /b = 0,5/0,1 + 5 m^3 / h [21]

By analyzing this equation we can conclude that for a sudden variation of the flow, for example from 5 to 10 m/h, the water level in the lake suffers an exponential change in the circumstances in which the banks are considered vertical and waterproof.

Along with the water flow that feeds the lake, and the water flow leaving the lake, which can be relatively easily monitored in accordance with water legislation, a number of substances affect the water quality of lakes, which can disrupt the ecosystem. By their nature, the polluting substances generally can be:

- of a physical nature (solid substances, radioactive substances, thermal pollution);
- of a chemical nature (hydrocarbons, carbon derivatives, sulfur, nitrogen, plastics, synthetic resins, pesticides, synthetic organic compounds, fluorides, heavy metals, metal oxides, fermentable organic material, inorganic chemicals, etc.);

- of a biological nature (organic manure, pathogens, bacteria, viruses, etc.)

Therefore the entries must be monitored in order to establish measures in case the trend becomes worse, resulting in a change in equilibrium with negative implications for the ecological balance.

In a liquid medium the equilibrium states are characterized by a uniform spatial distribution of each fluid properties, so that every element within it is in equilibrium with the surrounding elements, and therefore if from the starting point there is a non-uniform distribution of the properties, there will be an exchange of properties between the adjacent elements towards achieving uniform properties within the fluid.

If we look at only one parameter of water quality, for example, the particles in suspension which are usually deposited at the entry in the reservoir, depending on the speed of the water and the density of the particle, in this situation we can discuss about their sedimentation. Sedimentation conditions are known and will shall not go into further details.

At the discharge into the lakes of the waters bearing various substances there is a dilution of the concentration of incoming water, as a result of mixing of the water in the lake, which theoretically has very small concentrations of chemical substances.

From the point of view of a cybernetic system, the composition structure of the lake water, the inlet and outlet water in the ecosystem is largely influenced by a series of disturbing elements, such as precipitation, wind and water, as well as a series of aquatic reactions which influence the chemical composition of the ecosystem (Figure 3).

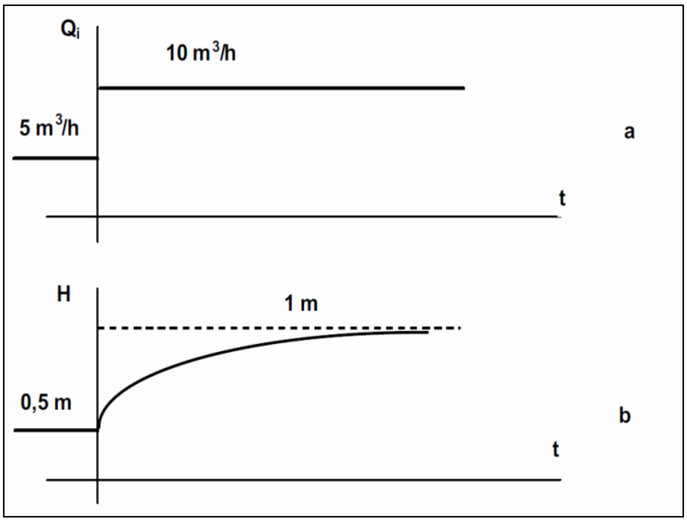

Figure 3. Dynamic characteristic of a liquid accumulation process:
a – time variation of the inflow; b – time variation of the level.

Generally the chemical reactions in the aquatic environment are of three types (Roman 1980):

- oxidation-reduction reactions;
- acid-base type reactions;
- complex reactions between organic compounds and metal compounds, also called metal-organic coordination reaction.

All of these processes can be carried out in the same water system, such as, for example, a lake or a stationary deeper water. In this ecosystem we can establish two chemically distinct zones of namely:

- anaerobic zone, located in the deep basin of water, where the dissolved oxygen level is very low;
- superficial area, where is an exchange of oxygen and CO_2 with the atmosphere, basically we can talk about an area of dissolved oxygen in large quantities.

In the top layer of water, where sunlight reaches, there are a series of oxidation reactions, as previously mentioned, but at the same time there are also processes of photosynthesis which occur when oxygen is released, according to the reaction:

$$CO_2 + H_2O \rightarrow \text{photosynthesis} \rightarrow CH_2O + O_2$$

In contrast, in the depth, the processes which dominate the area are those of reduction due to the fact that in general the environmental conditions are anaerobic. In this area is produced an exchange of materials with the sediments found at the bottom of lakes.

In continental deep waters there are reducing components, such as ammonia hydrogen sulfide (H_2S), ferrous ions and Mn ions. In contrast in surface areas in particular, there is a concentration of dissolved oxidizing elements, as for example, nitrate ions (NO_3), sulphate (SO_4), carbonate (CO_3), and other oxidizing agents in suspension (Roman 1980).

For the most part of the year the two areas: the shallow and the deep do not mix, primarily due to different density of layers. Different density is due to the exposure to the sun of the surface layer. The phenomenon is called thermal stratification and basically is a physical barrier of separating the layers.

Conclusions. Considering the aquatic ecosystem as a cybernetic system is a very complex problem and it involves a large volume of measurements and analysis. The cybernetic system in the definition should allow conducing processes, respectively the influencing parameters involved in the process to reach equilibrium or an ideal or optimal state of the ecosystem.

If a number of parameters, such as input and output flows can be tracked and influenced by using weirs or thresholds through which flows are subject to change, the lack of oxygen in the water or the presence of chemical compounds in water, which significantly affect the quality of the ecosystem, is a more complicated problem and solving it requires time and investment.

A major issue is raised by external perturbations that are not predictable and cannot be managed or influenced. We cannot anticipate or predict the volume and exact timing of precipitation. If you have large amounts of short-term rainfall it means that we have increased the input flow, a flow accompanied by significant quantities of silt or suspended matter that will affect the ecosystem and in this case we do not have solutions for correction or influencing.

Along with the water level in the ecosystem, the amount of oxigen dissolved in water is a very important parameter for the good functioning of an ecosystem. In this case we can continuously monitor the quantity of dissolved oxygen, but we can influence only slightly increase of the the amount of oxygen. Of course, the placement ecosystem water oxygenation systems (semi-aquatic propellers), air insufflation etc, is not a solution, mainly because of the high costs of energy. It is however possible the installation of wind or photovoltaic cells to provide energy needed for such aeration systems especially during hot seasons, knowing that the degree of dissolution of oxygen in water is directly influenced by water temperature, as the water temperature, namely the higher the water temperature, the lower the amount of oxygen that can be dissolved in the water.

We can predict only approximatively the temperature variation and we can neither influence it. Water temperature influences the existence of rising water currents that can provide transport of dissolved oxygen to the bottom. Also the water temperature, namely the solar radiation influences the evaporation of water from the lake, which is to be taken into account in determining the constant of the integration formula for the study of variation of water depth depending on the water flow.

It should be noted that the lake cannot be regarded as a vessel with vertical walls. This greatly complicates the flow correlation with depth because the formula for calculation of the water surface of the lake is an important parameter, which theoretically in the example we considered to be a constant, but in reality it is variable depending on the nature of the land and water depth.

The information presented in these findings proves the immense complexity of an aquatic ecosystem and that is extremely hard to be able to infuence the large number of parameters that characterize an ecosystem’s quality.

We can also conclude that an aquatic ecosystem and is extremely vulnerable to the influence of human activities and changing weather conditions. The global warming which we are unfortunately witnessing adversely affects the aquatic ecosystems.

References

Roman P., 1980 [Introducere în fizica poluării fluidelor]. Scientific and Encyclopedic Publishing House, Bucharest. [in Romanian].

Rusu T. A., 2010 [Sisteme informaţionale privind monitorizarea şi gestiunea factorilor de mediu]. UTPRESS Publishing house, ISBN- 978-973-662-524-4 [in Romanian].

Rusu T. A., 2010 [Cercetări privind sisteme de monitorizare a factorilor de mediu]. PhD thesis [in Romanian].

Received: 19 January 2015. Accepted: 26 February 2015. Published online: 31 March 2015.

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How to cite this article:
Rusu A. T., Pădureţu S., Rusu T., 2015 Research on the application of a dynamic mathematical model to the monitoring of an aquatic ecosystem. Ecoterra 12(1):83-88.